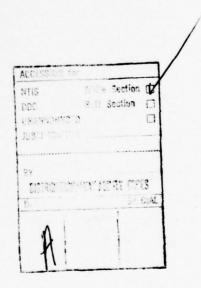


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INTRODUCTION

The relationship between free-air gravity anomalies and sea-floor bathymetry is one of the most important problems in marine gravity. For purely practical reasons, deciphering the nature of such a relationship is vital to predicting quantitatively the gravity field in oceanic areas in which only bathymetry is known. Closely related to this practical problem is the question of the physical basis for any observed or proposed dependence of gravity upon topography. Such a question bears on the mechanisms for creation and evolution of oceanic crust and lithosphere and on the possible interaction of the lithosphere with motions in the asthenosphere.

Marine gravity and sea-floor topography cannot be related by a simple mathematical expression (e.g., a line) that has validity in all oceanic environments. A recent synthesis of submarine gravity data and bathymetry by Woolard and Daugherty (1970) demonstrated the necessity to divide oceanic regions by tectonic types and the difficulty in simply relating free-air anomaly and depth even within groups of tectonically similar environments. Degree averages of marine gravity anomalies and depths within selected regions were also correlated with varying degrees of success by Watts and Talwani (1974), Sclater et al. (1975), and Watts (1976).

A necessary step in improving what heretofore have generally been strictly empirical attempts to derive a rule relating gravity and bathymetry over a selected region is to take into proper account the findings of plate tectonic studies of the oceans. At mid-ocean ridges, the lithosphere is thin, perhaps no thicker than the crust (Francis and Porter, 1973; Solomon and Julian, 1974; Orcutt et al., 1975; Rosendahl et al., 1976), and sea floor topography except at the shortest wavelengths is isostatically compensated with a shallow compensation depth (Dorman, 1975; McKenzie and Bowin, 1976; Watts and Cochran, 1977). Sea floor topography created by mid-plate volcanic activity is compensated much deeper because of the substantially

thicker lithosphere, and the compensation is regional (Vening Meinesz, 1941), involving flexure of the oceanic plate (Walcott, 1970; Watts and Cochran, 1974; Watts et al., 1975), the effective elastic thickness of which increases with sea floor age (Caldwell and Turcotte, 1978). At trench systems, topography is not isostatically compensated owing to the large dynamic forces associated with lithosphere subduction (e.g., Vening Meinesz, 1954). Still controversial are explanations for frequent correlations in very long wavelength gravity and topographic anomalies in the oceans (Menard, 1973; Anderson et al., 1973; Weissel and Hayes, 1974; Sclater et al., 1975; Marsh and Marsh, 1976; Cochran and Talwani, 1977; Watts, 1978), with asthenospheric flow the most exciting but still unproven hypothesis.

This report lays the mathematical framework for a simple but general relation between topography and gravity in two dimensions for stable ocean basins. The conceptual framework for the relation is based on flexure theory for thin elastic plates loaded from above. The evolution with sea-floor age of lithospheric temperature and rheology is abstracted to an effective elastic layer thickness which grows with plate age. In order to derive gravity from topography using this relation, both the lithospheric age and the age of any more recently superposed volcanic constructs (islands, seamounts, aseismic ridges) must be known. Guidelines are given for estimating these ages. Finally, the quantification of the bathymetry-gravity relation using data from the central Pacific basin, work being conducted during the second year of this contract research, is briefly described.

We seek a relation between bathymetry and gravity in stable ocean basins. We shall work explicitly in spatial dimensions, rather than use one- or two-dimensional wavenumber representations, so as to permit consideration of regions of arbitrary geometry and spatial extent.

We regard the sea-floor topography as composed of three parts: (1) the long-wavelength deepening of the sea floor with age due to thermal contraction of the lithosphere (Sclater et al., 1971); (2) the volcanic topography emplaced as a load on top of the lithosphere; and (3) the lithospheric response to that load. We assume that the effect of (1) on the bathymetry may be removed with a suitable age-depth relation (Sclater et al., 1975; Cochran and Talwani, 1977); its effect on gravity is negligible far from ridges (Lambeck, 1972).

Let the lithosphere be modeled as a thin, spherical, elastic shell of thickness T, Young's modulus E, and Poisson's ratio ν , overlying a fluid interior of density ρ_{m} . Let R be the radius to the midplane of the shell, let g be the gravitational acceleration at radius r = R - T/2, and let D be the flexural rigidity of the shell

$$D = \frac{ET^3}{12(1 - v^2)} \tag{1}$$

Consider a uniform vertical load ${\rm q}$ (force per unit area) applied to a circular unit area on the surface of the shell. The vertical deflection (negative if downward) of the lithosphere is given by

$$w = \frac{q}{2\pi (ET/R^2 + \Delta \rho \ g)} \quad \text{kei } \xi$$
 (2)

(Brotchie and Sylvester, 1969; Brotchie, 1971), where ξ is the distance from the load center, normalized by the radius of

relative stiffness

$$\ell = \left(\frac{D}{ET/R^2 + \Delta \rho \ g}\right)^{1/4} ; \qquad (3)$$

kei is a Bessel-Kelvin function of order zero (Abramowitz and Stegun, 1964); and $\Delta\rho=~\rho_m^{}-~\rho_w^{}$ where $\rho_w^{}$ is the density of seawater if the load is applied at the sea bottom and zero if the load is applied on land. Note that (2) is equivalent to the expression (Brotchie and Sylvester, 1969) for the response to a point force p since p = $q\pi d^2 \ell^2$ where ℓ = d is the radius of the circular unit area. Equations (2) and (3) may be simplified because for oceanic lithosphere ET/R² ~ 101 and $\Delta\rho$ g = 2 x 10³ in c.g.s. units, so $\Delta\rho$ g >> ET/R² and

$$w = \frac{q}{2\pi\Delta\rho \ g} \text{ kei } \xi. \tag{4}$$

We may generalize (4) to a distributed load. Let q(x,y) be the force per unit area exerted on the lithosphere by topography. Then the deflection w is given by

$$\mathbf{w}(\mathbf{x},\mathbf{y}) = \frac{1}{2\pi\Delta\rho \ \mathbf{g}} \iint \mathbf{q}(\mathbf{x}',\mathbf{y}') \ \mathrm{kei} \ (\mathbf{r}'/\ell) \ \mathrm{d}\mathbf{x}'\mathrm{d}\mathbf{y}' \tag{5}$$

where $r' = [(x' - x)^2 + (y' - y)^2]^{1/2}$. Equation (5) may be thought of as the double convolution of q with the flexural response function

$$\Phi(\mathbf{x},\mathbf{y}) = \frac{1}{2\pi\Delta\rho} \operatorname{kei}\left[\frac{(\mathbf{x}^2 + \mathbf{y}^2)^{\frac{1}{2}}}{\ell}\right]$$
 (6)

which in turn is a function of the local flexural ridigity D, or equivalently the effective elastic lithospheric thickness T, through

$$\ell = \left(\frac{D}{\Delta \rho \ g}\right)^{\frac{1}{4}} .$$

A single convolution of (two-dimensional) load with response function was also used by Roufosse and Parsons (1977) in their study of the Hawaiian ridge. We regard & as a function of x and y, dependent in a predictable fashion on the age of the lithosphere (Caldwell and Turcotte, 1978) and on the age of the local load. If the load was emplaced in a time short compared to the lithosphere age and if viscous relaxation of stress subsequent to initial lithospheric flexure has been minimal, then & is simply a function of the time difference between plate age and load emplacement age.

Unfortunately, the load q(x,y) is not known a priori but rather the ocean floor elevation h(x,y) with respect to some arbitrary datum is known. As noted above, h contains contributions from both the topographic load and the lithospheric response. We may resolve this difficulty, however, by iteration. Assume initially that $q(x,y) = \Delta \rho' gh(x,y)$ where $\Delta \rho' = \rho_t - \rho_w$ and ρ_t is the density of the major units constituting the topographic variations. Then use (5) to calculate w(x,y). Now set $q(x,y) = \Delta \rho' g(h-w)$. Recalculate w from (5), and repeat the last two steps until q and w converge to a steady solution.

Once a self-consistent decomposition of h into load topography $q/\Delta\rho$ 'g plus plate deflection w is achieved, the calculation of gravity is straightforward. Two terms contribute to the gravity: (i) the attraction of the topography and (ii) the deflection of the Moho and of any other density contrast interfaces within the lithosphere. Any density contrast between the base of the lithosphere and the asthenosphere would also contribute to (ii), but this contribution is probably negligible. For both (i) and (ii) the gravity anomaly may be written in the form of that due to a surface mass distribution $\sigma(x,y)$

$$\Delta g(x,y,z) = Gz \iint \frac{\sigma(x',y')}{[(x'-x)^2+(y'-y)^2+z^2]^3/2} dx'dy'(7)$$

where G is the gravitational constant and z> 0 is the vertical distance between the observation point (x,y,z) and the horizontal plane on which σ is evaluated. For the topographic contribution, $\sigma = \Delta \rho'$ h on the plane of the topographic datum (h=0). For the plate deflection contribution, $\sigma = \Delta \rho$ w on the plane corresponding to mean Moho depth.

An independent check on any bathymetry-gravity model is the lithospheric stress. The stress field, as inferred from lithospheric earthquake mechanisms, say, may be a strong function of the load (Rogers and Endo, 1977). The stress may also be calculated directly once the load q(x,y) is determined. For the single load q applied to a circular unit area, the bending moments M and stress resultants N expressed as components along the polar angle ϕ and azimuthal angle θ with respect to the load center are (Brotchie, 1971)

$$\begin{split} \mathbf{M}_{\varphi} &= -\frac{\mathbf{q}}{2\pi} \; (\text{ker } \xi \; + \frac{1-\nu}{\xi} \; \text{kei'} \xi) \\ \mathbf{M}_{\theta} &= -\frac{\mathbf{q}}{2\pi} \; (\nu \, \text{ker } \xi \; + \frac{1-\nu}{\xi} \; \text{kei'} \xi) \\ \mathbf{N}_{\varphi} &= \frac{\mathbf{q} \ell^2 \mathrm{ET/R}}{2\pi \xi \mathrm{D}} \; \left(\frac{1}{\xi} \; + \; \text{ker'} \; \xi \; \right) \\ \mathbf{N}_{\theta} &= -\mathbf{N}_{\varphi} \; - \frac{\mathrm{ETw}}{\mathrm{R}} \; = -\frac{\mathbf{q} \mathrm{ET/R}}{2\pi} \; \left[\frac{\ell^2}{\xi \mathrm{D}} \; \left(\frac{1}{\xi} \; + \; \text{ker'} \; \xi \right) \; + \; \text{kei } \xi \right] \end{split}$$

where ker is another Bessel-Kelvin function of order zero and where prime denotes first derivative.

The horizontal stress components within the plate at radius r are given by

$$\sigma_{\dot{\phi}} = \frac{N_{\dot{\phi}}}{T} + \frac{12M_{\dot{\phi}}}{T^3} (r - R)$$

$$\phi_{\dot{\theta}} = \frac{N_{\dot{\theta}}}{T} + \frac{12M_{\dot{\theta}}}{T^3} (r - R) .$$
(9)

For a distributed load, q(x,y), suppose θ is measured clockwise from the +y direction, and let the x and y axes point E and N, respectively. Then

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} (x,y) = \iint \frac{R}{Z} \begin{bmatrix} \sigma_{\phi} & 0 \\ 0 & \sigma_{\theta} \end{bmatrix} (x' - x, y' - y) \frac{R}{Z} dx' dy'$$
(10)

where

$$\frac{R}{-} = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$$

$$\theta = \tan^{-1} \frac{x - x'}{y - y'}$$

$$\phi = \frac{\left[(x' - x)^2 + (y' - y)^2 \right]^{\frac{1}{2}}}{R}$$

and where σ_{φ} and σ_{θ} are determined from $\,\,q\,\,$ using (8) and (9) with ξ = R\$\phi/\ell\$.

SOME PRACTICAL CONSIDERATIONS

The theoretical relationship between topography and gravity outlined above can be applied to any finite area over which bathymetry and age are specified as functions of position. We are currently applying these concepts to regions in which degree averages of ocean floor depth (corrected for mean age) and free air gravity are known. Implementation of the theory as a usable algorithm requires estimates of volcanic load ages and of the evolution of the flexural response function with age, and calculational schemes for evaluating the convolution integrals in (5) and (7).

Some care must be exercised in deciding the age of a volcanic construct relative to the age of the surrounding seafloor. For islands this is not a serious problem as the exposed or cored (if a coralline island) bedrocks can be dated. For seamounts some simple rules will be helpful. One such rule is that an inactive topographic feature will sink with respect to sea level at the rate of its surrounding abyssal sea floor, a rate which is associated with thermal contraction of the lithosphere and which is a well known function of ocean-floor age (Sclater et al., 1971). On this basis, for instance, the Ninetyeast ridge can be shown to have been generated at the southeast Indian ridge (Sclater and Fisher, 1974). Much of the ocean floor topography, in fact, was apparently generated at or near ridge crests on very young oceanic lithosphere (McKenzie and Bowin, 1976).

The flexural rigidity D and the effective elastic plate thickness T are known to vary from values in the range $3 \times 10^{27} - 2 \times 10^{29}$ dyne-cm and 3 - 13 km, respectively, near mid-ocean ridges (Watts and Cochran, 1977; Detrick and Watts, 1978) to values in the range 3×10^{29} to 10^{30} dyne-cm and 15 - 30 km, respectively, for older lithosphere (Walcott, 1970; Watts and Cochran, 1974; Watts et al., 1975; 1976; Caldwell et al., 1976). The assumption that T scales as an isotherm depth is consistent with the data (Caldwell and Turcotte, 1978), so that a standard oceanic plate thermal

model (Parsons and Sclater, 1977) and a few good measures of D and T are sufficient to define T as a function of lithosphere age minus load age.

Evaluation of the necessary convolution integrals has been conducted as follows. Equation (5) is evaluated by dividing each area element over which mean elevation is known (e.g., degree squares) into subelements and treating the load on each subelement as an equivalent load on a unit area in the subelement center. Computation time is saved by noting that the contribution to w(x,y) from the load q(x',y') depends only on the scalar distance from (x',y') to (x,y). Bessel-Kelvin functions are evaluated using a FORTRAN subroutine package (COMBES), written at the MIT Information Processing Center, for calculating Bessel functions of complex argument and order. Equation (7) is evaluated using the Taylor series algorithm of Morrison (1976) for calculating the gravitational potential from a surface density distribution specified over a latitude-longitude grid, and a centered finite difference to obtain the radial derivative of the potential.

INTENDED APPLICATIONS

The bathymetry-gravity algorithm is currently being applied to the central Pacific basin, a region chosen because the gravity, bathymetry and lithospheric age are all well known and because there are superposed volcanic loads on the plate of various younger ages. The gravity for the region is taken from the degree averages of Watts and Leeds (1977). Bathymetry is taken from a DMA compilation provided by AFGL (T.P. Rooney, personal communication, 1978). Plate age is taken from standard magnetic anomaly information (Pitman et al., 1974).

Results from the bathymetry-gravity analysis for the central Pacific, and an assessment of its general validity, will be given in the next Scientific Report.

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LIST OF CONTRIBUTING SCIENTISTS (1 November 1976 - 31 October 1977)

- S.C. Solomon, Principal Investigator, part-time
- J. Chaiken, Graduate Research Assistant, part-time

LIST OF PREVIOUS CONTRACTS AND PUBLICATIONS (1 November 1976 - 31 October 1977)

None